

Multiphoton controllable transport between remote resonators

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We develop a novel method for multiphoton controllable transport between remote resonators. Specifically, an auxiliary resonator is used to control the coherent long-range coupling of two spatially separated resonators, mediated by a coupled-resonator chain of arbitrary length. In this manner, an arbitrary multiphoton quantum state can be either transmitted through or reflected off the intermediate chain on demand, with very high fidelity. We find, on using a time-independent perturbative treatment, that quantum information leakage of an arbitrary Fock state is limited by two upper bounds, one for the transmitted case and the other for the reflected case. In principle, the two upper bounds can be made arbitrarily small, which is confirmed by numerical simulations.

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I. INTRODUCTION

The realization of quantum information transport between remote parties is of central importance for any scalable quantum information processing. To this end, one straightforward approach is to use quantum channels to connect these spatially separated parties. Owing to high speed transmission and negligible interaction with the environment, in the form of flying qubits, photons are the natural candidates for long-distance quantum communication [1–7]. Alternatively, for short-distance quantum communication inside a quantum computer, the majority of promising channels rely upon the use of solid-state-based devices, including nuclear spins in nuclear magnetic resonance (NMR) [8, 9], electron spins of nitrogen-vacancy (NV) colour centers in diamond [10–15], and flux qubits in superconductors [16–19]. Moreover, coupled-resonator arrays (CRAs), being currently explored in various physical systems such as superconducting transmission line resonators [20–24], toroidal microresonators [25–28] and plasmonic nanoparticle arrays [29], have been attracted much attention in recent years. A particular advantage of such arrays is the full addressability of individual resonators, which allows each of the resonators to act as a quantum network node [30–32]. Indeed, utilizing these CRAs of having the same fundamental hardware to process quantum information can also avoid a quantum interface between the quantum register and the quantum channel.

In addition to offering an effective platform to simulate quantum many-body phenomena such as Mott-superfluid and topological effects [33–36], the CRAs have been previously considered for controllable transport of photons by making use of the photon-atom scattering [22, 37–41]. Despite such substantial developments, prior work on controllable photon transport has typically focused upon either single photons [22, 38, 42–44] or nearby CRAs [45–49]. However, the ability to transport multiphoton quantum states is a key requirement for encoding a high-dimensional Hilbert space, which is applicable, for example, to universal quantum computation [50–52]; at the same time, quantum information stored in the multiphoton fields needs to be controllably transported between two distant quantum registers to carry out quantum network operations. For these reasons, developing a quantum channel capable of controllably transporting multiphoton states is thus of both fundamental and practical importance.

In this paper, we propose and analyze a multiphoton controllable transport protocol, where we use an auxiliary resonator coupled to one resonator of a coupled-resonator chain, which serves as a quantum channel to connect two remote resonators. The physical essence underlying our method is that this auxiliary resonator is employed to control the coherent long-range interaction between the two boundary resonators. To be specific, in the case when the auxiliary resonator is absent, the two boundary resonators could be only coupled to a specific eigenmode of the intermediate chain, within the weak-coupling regime. In this case, the time evolution can swap arbitrary bosonic quantum states of the two boundary resonators, yielding an effective photon transport channel (EPTC). On the contrary, when the auxiliary resonator is coupled to the intermediate chain, the specific eigenmode could be split, such that the two boundary resonators are decoupled from the intermediate chain in the large-detuning limit. Photons are therefore

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reflected back, remaining unchanged. As opposed to prior work, the proposed model is capable of controlling the coherent transport of an arbitrary multiphoton quantum state between two distant resonators over an arbitrarily long range.

II. PHYSICAL MODEL AND EFFECTIVE HAMILTONIAN

The basic idea of our protocol is schematically illustrated in Fig. 1(a). To begin, we consider a quantum channel consisting of a coupled-resonator chain of N resonators, with a Hamiltonian

$$H_0 = \kappa \sum_{i=1}^{N-1} \left(c_i^\dagger c_{i+1} + \text{H.c.} \right), \quad (1)$$

where c_i (c_i^\dagger) represents the annihilation (creation) operator acting on the resonator i and obeys a boson commutation relation $[c_i, c_j^\dagger] = \delta_{ij}$, and κ is the coupling constant between the nearest-neighbor resonators. Two distant resonators, labelled 0 and $N+1$, are coupled to the ends of the quantum channel, and the interaction Hamiltonian is correspondingly given by

$$V_1 = g_0 \left(c_0^\dagger c_1 + c_{N+1}^\dagger c_N + \text{H.c.} \right), \quad (2)$$

where g_0 is the coupling strength between the two boundary resonators and the intermediate chain. In order to control the multiphoton coherent transport, we introduce an auxiliary control resonator, labelled $N+2$, to interact with the m -th resonator of the quantum channel through

$$V_2 = J_0 \left(c_{N+2}^\dagger c_m + \text{H.c.} \right), \quad (3)$$

with a coupling strength J_0 . Here, we have assumed that all resonators have a common frequency ω , and the system is transformed into a frame rotating at ω . Therefore, the total Hamiltonian governing the system is $H_T = H_0 + V_1 + V_2$.

Upon following an orthogonal transformation [55, 56],

$$f_k = \sqrt{\frac{2}{N+1}} \sum_{i=1}^N \sin \frac{ik\pi}{N+1} c_i, \quad (4)$$

the Hamiltonian H_0 is diagonalized to

$$H_0 = \sum_{k=1}^N \varepsilon_k f_k^\dagger f_k, \quad (5)$$

where $\varepsilon_k = 2\kappa \cos[k\pi/(N+1)]$. Substituting Eq. (4) into Eqs. (2) and (3), V_1 and V_2 are likewise transformed to

$$V_1 = \sum_{k=1}^N g_k \left[c_0^\dagger f_k + (-1)^{k-1} c_{N+1}^\dagger f_k + \text{H.c.} \right], \quad (6)$$

and

$$V_2 = \sum_{k=1}^N J_k \left(c_{N+2}^\dagger f_k + \text{H.c.} \right), \quad (7)$$

respectively. Here, we have defined

$$g_k = g_0 \sqrt{\frac{2}{N+1}} \sin \frac{k\pi}{N+1}, \quad (8)$$

and

$$J_k = J_0 \sqrt{\frac{2}{N+1}} \sin \frac{mk\pi}{N+1}. \quad (9)$$

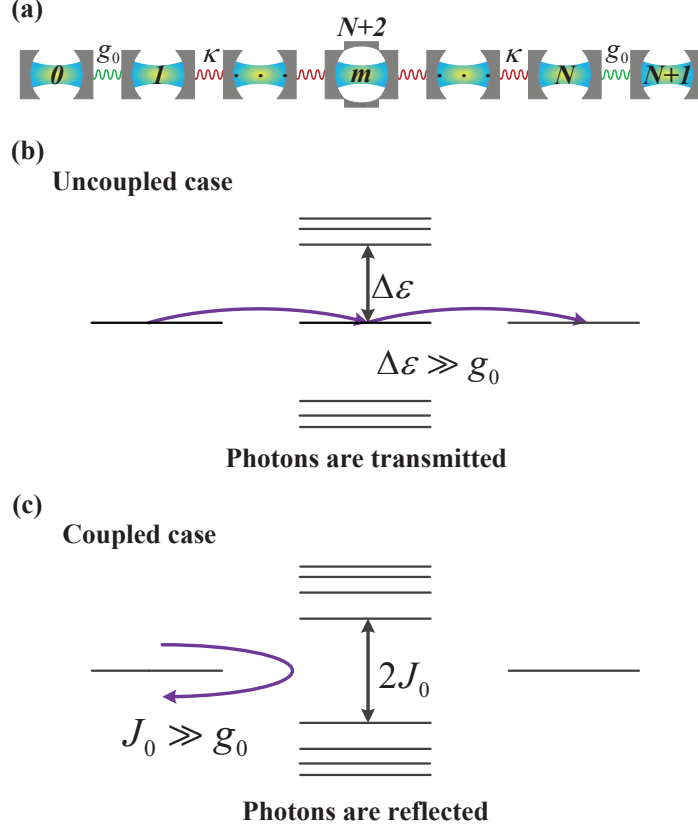


FIG. 1: (color online) (a) Two remote resonators, labelled 0 and $N + 1$, are coupled to the ends of a coupled-resonator chain of N resonators with g_0 , the coupling strength between the two boundary resonators and the intermediate chain, and κ , the coupling strength between the intrachain elements. To achieve multiphoton transport, we introduce an auxiliary resonator, labelled $N + 2$, to interact with the m -th resonator of the intermediate chain, with strength J_0 . (b) In uncoupled case of $J_0 = 0$, the intermediate chain could, under the assumption that $g_0 \ll \kappa$, coherently couple the two spatially separated resonators, such that photons are transported between the two boundary resonators after the time evolution. (c) However, when $g_0 \ll J_0 \ll \kappa$, the boundary resonators are decoupled from the intermediate chain owing to the large detunings. In this case, photons are reflected back and remains unchanged.

By restricting our attention to odd N , it yields the existence of a single zero-energy mode in the intermediate chain corresponding to $k = z \equiv (N + 1)/2$, such that this mode is in resonance with the two boundary resonators as well as with the auxiliary resonator. Under the assumption that g_0 and J_0 are much smaller than κ , off-resonant couplings to the boundary resonators and to the auxiliary resonator can be neglected owing to $\{g_z, J_z\} \ll |\epsilon_{z\pm 1} - \epsilon_z|$ [57]. As a result, the full evolution dynamics is reduced to an effective model in which only the two boundary resonators, the auxiliary resonator and the zero-energy mode are involved, and accordingly H_T is approximated as an effective Hamiltonian

$$H_{\text{eff}} = g_z \left[c_0^\dagger f_z + (-1)^{z-1} c_{N+1}^\dagger f_z + \text{H.c.} \right] + J_z \left(c_{N+2}^\dagger f_z + \text{H.c.} \right), \quad (10)$$

which can be used to make a multiphoton quantum switch.

III. MULTIPHOTON CONTROLLABLE TRANSPORT AND FIDELITIES

If the auxiliary resonator is uncoupled to the intermediate chain, $J_0 = 0$, the effective Hamiltonian becomes

$$H_{\text{eff}} = g_z \left[c_0^\dagger f_z + (-1)^{z-1} c_{N+1}^\dagger f_z + \text{H.c.} \right]. \quad (11)$$

In this case, the two boundary resonators are coherently coupled by means of the zero-energy mode. In the Heisenberg picture, a straightforward calculation yields

$$c_0^\dagger(t) = c_0^\dagger + \frac{1}{2} \left[-1 + \cos(\sqrt{2}g_z t) \right] \left[c_0^\dagger + (-1)^{z-1} c_{N+1}^\dagger \right] + \frac{i \sin(\sqrt{2}g_z t)}{\sqrt{2}} f_z.$$

Choosing the evolution time $t = \tau \equiv \pi/\sqrt{2}g_z$ gives

$$c_0^\dagger(\tau) = (-1)^z c_{N+1}^\dagger, \quad (12)$$

and in a similar manner, we have

$$c_{N+1}^\dagger(\tau) = (-1)^z c_0^\dagger. \quad (13)$$

Eqs. (12) and (13) exhibit that the evolution for a specific time behaves as a swap operation between the two boundary resonators, as shown in Fig. 1(b). However, this zero-energy mode could, in the case when the auxiliary resonator is coupled to the intermediate channel, be split into two new modes separated by an energy gap $2J_z$. It follows, on ensuring $g_0 \ll J_0$, that the two boundary resonators are significantly detuned from the new modes if m is odd, and thus are decoupled from the intermediate channel. In this case, the time evolution is referred to as an identity operation, leading to

$$c_0^\dagger(t) = c_0^\dagger, \quad (14)$$

and

$$c_{N+1}^\dagger(t) = c_{N+1}^\dagger. \quad (15)$$

Let us consider the controllable transport of an arbitrary Fock state $|n\rangle$, for example, from the left resonator to the right resonator. The more general treatment of an arbitrary superposition of Fock states are presented in the Appendix. Such a Fock state can be generated using a nonlinear quantum system as an intermediary between a classical radiation field and the resonator [53]. We start with an initial state of the total system,

$$|\Phi(0)\rangle = |n\rangle_0 |\mathbf{0}\rangle |0\rangle_{N+1} = \frac{(c_0^\dagger)^n}{\sqrt{n!}} |0\rangle_T, \quad (16)$$

where $|\mathbf{0}\rangle = |0\rangle_1 \cdots |0\rangle_N |0\rangle_{N+2}$, and $|0\rangle_T = |0\rangle_0 |\mathbf{0}\rangle |0\rangle_{N+1}$ is the vacuum state of all resonators. Under the time evolution, the total system freely evolves into a final state,

$$|\Phi(t)\rangle = \frac{[c_0^\dagger(-t)]^n}{\sqrt{n!}} |0\rangle_T. \quad (17)$$

In the uncoupled case when $J_0 = 0$, according to Eq. (12), the final state for time τ is $|\Phi(\tau)\rangle = (-1)^{nz} |0\rangle_0 |\mathbf{0}\rangle |n\rangle_{N+1}$, which means that all photons are simultaneously transported from the left resonator to the right resonator as desired. On the contrary, in the coupled case of $g_0 \ll J_0 \ll \kappa$, the final state becomes $|\Phi(t)\rangle = |\Phi(0)\rangle$ according to Eq. (14), thus these photons are reflected back, remaining unchanged.

In order to characterize the quality of our protocol, we employ two fidelities with one transmission fidelity, $F_t = \langle n | \rho_{N+1}(\tau) | n \rangle$, and one reflection fidelity, $F_r = \langle n | \rho_0(\tau) | n \rangle$. Here, $\rho_0(\tau)$ and $\rho_{N+1}(\tau)$ are the reduced density matrices of the resonators 0 and $N+1$, respectively, at time $t = \tau$. Together with an $(N+1) \times (N+1)$ coupling matrix A , the total Hamiltonian can be compactly expressed as

$$H_T = \sum_{i,j=0}^{N+2} A_{ij} c_i^\dagger c_j, \quad (18)$$

where A_{ij} is the coupling strength between two resonators i and j . Applying the Heisenberg equation of motion, $c_i(t) = i[H_0, c_i(t)]$, one finds that

$$c_i(t) = \sum_{j=0}^{N+2} M_{ij} c_j, \quad (19)$$

with $M = \exp(-iAt)$ being a unitary evolution matrix. To calculate the two fidelities F_t and F_r , more explicitly, we rewrite $c_i(t)$ of Eq. (19) as

$$c_0(t) = M_{0,\mu}c_\mu + \sqrt{\delta_\mu}c_{\delta_\mu}, \quad (20)$$

where $\delta_\mu = 1 - |M_{0,\mu}|^2$ and $\mu = 0, \dots, N+2$. Here, we define a collective mode c_{δ_μ} as a normalized linear combination of all modes apart from c_μ , resulting in $[c_\mu, c_{\delta_\mu}^\dagger] = 0$ and $[c_{\delta_\mu}, c_{\delta_\mu}^\dagger] = 1$. In combination with Eqs. (17) and (20), we have

$$|\Phi(t)\rangle = \sum_{r=0}^n f_\mu(r, n) |r\rangle_{\delta_\mu} |n-r\rangle_\mu, \quad (21)$$

where

$$f_\mu(r, n) = \sqrt{C_n^r} M_{0,\mu}^{n-r} \delta_\mu^{r/2}, \quad (22)$$

$$C_n^r = \frac{n!}{r!(n-r)!}, \quad (23)$$

and $|r\rangle_{\delta_\mu}$ is a Fock state of the collective mode c_{δ_μ} . The corresponding density matrix is

$$\begin{aligned} \rho(t) &= |\Phi(t)\rangle \langle \Phi(t)| \\ &= \sum_{r,r'=0}^n f_\mu(r, n) f_\mu^*(r', n) |n-r\rangle_\mu \langle n-r'| \otimes |r\rangle_{\delta_\mu} \langle r'|, \end{aligned} \quad (24)$$

and then, by tracing out the variables of the collective mode, the reduced density matrix of the resonator μ is calculated as

$$\rho_\mu(t) = \sum_{r=0}^n |f_\mu(r, n)|^2 |n-r\rangle_\mu \langle n-r|. \quad (25)$$

Thus, the transmission fidelity and the reflection fidelity are respectively given by

$$F_t = |f_{N+1}(0, n)|^2 = |M_{0,N+1}|^{2n}, \quad (26)$$

and

$$F_r = |f_0(0, n)|^2 = |M_{0,0}|^{2n}. \quad (27)$$

In Fig. 2 we plot the transmission fidelity F_t versus J_0/κ with $g_0/\kappa = 0.01$ [see Fig. 2(a)], or 0.005 [see Fig. 2(b)], as well as the reflection fidelity F_r . In the two cases, we assume that a Fock state $|n\rangle$ is initialized into the left boundary resonator, and further take $n = 2, 3$ and 5 as three special examples to simulate numerically. It is shown that as $J_0 = 0$, we obtain $F_t \simeq 1$ while $F_r \simeq 0$, implying that photons are transmitted from the left resonator to the right resonator. On the contrary, in the regime $g_0 \ll J_0 \ll \kappa$, photons are reflected back to the left resonator, according to $F_t \simeq 0$ whereas $F_r \simeq 1$. Hence, a Fock state could, by tuning the coupling strength J_0 , be transmitted forward or reflected backward at will.

IV. QUANTUM INFORMATION LEAKAGE

Having explicitly demonstrated the controllable eigenmode-mediated transport of multiphoton information, we now calculate the quantum information leakage by making use of perturbation theory. We begin by considering the evolution matrix of Eq. (19). In fact, the coupling matrix A is identical to the total Hamiltonian confined in a single-excitation subspace,

$$H_S = g_0(|0\rangle\langle 1| + |N+1\rangle\langle N| + \text{H.c.}) + \kappa \sum_{j=1}^{N-1} (|j\rangle\langle j+1| + \text{H.c.}) + J_0(|N+2\rangle\langle m| + \text{H.c.}), \quad (28)$$

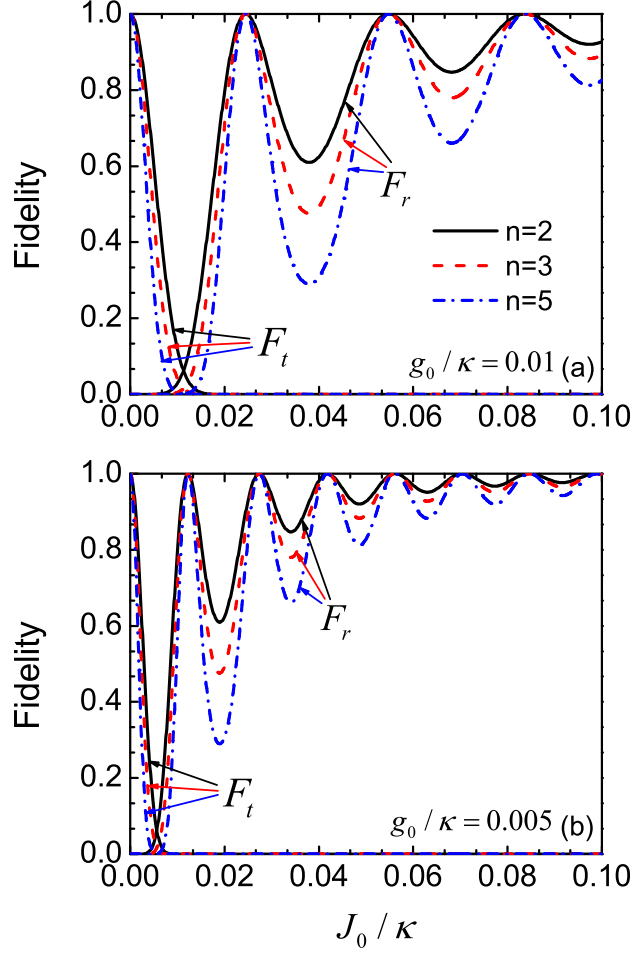


FIG. 2: (color online) Transmission fidelity F_t and reflection fidelity F_r as a function of J_0/κ with either (a) $g_0/\kappa = 0.01$ or (b) 0.005 for $n = 2, 3$ and 5 , corresponding to solid black lines, dashed red lines and dot-dashed blue lines, respectively. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

where $|\mu\rangle = c_\mu^\dagger|0\rangle_T$ ($\mu = 0, \dots, N+2$) represents the μ -th basis of this single-excitation subspace. As a consequence, the evolution matrix M is identical to $U(t) = e^{-iH_S t}$, so that $M_{\mu,\mu'} = \langle\mu|U(t)|\mu'\rangle$. By introducing an orthogonal transformation,

$$|k\rangle = \sqrt{\frac{2}{N+1}} \sum_{j=1}^N \sin \frac{jk\pi}{N+1} |j\rangle, \quad (29)$$

the Hamiltonian H_S is transformed to

$$H_S = \sum_{k=1}^N g_k \left[|0\rangle\langle k| + (-1)^{k-1} |N+1\rangle\langle k| + \text{H.c.} \right] + \sum_{k=1}^N \varepsilon_k |k\rangle\langle k| + \sum_{k=1}^N J_k (|N+2\rangle\langle k| + \text{H.c.}). \quad (30)$$

Based upon such a Hamiltonian, we can obtain the leakage of quantum information for the uncoupled case of $J_0 = 0$, and the coupled case of $g_0 \ll J_0 \ll \kappa$, respectively, as we will show in the following.

Let us first focus upon the former case, where the total system can be thought of as an EPTC being perturbatively coupled to a fictitious bosonic environment in the limit $g_0 \ll \kappa$. The corresponding Hamiltonian could be divided into three parts, $H_S = H_E + H_z + V_z$. H_E features the Hamiltonian for the EPTC:

$$H_E = g_z \left[|e_1\rangle\langle e_2| + (-1)^{z-1} |e_3\rangle\langle e_2| + \text{H.c.} \right], \quad (31)$$

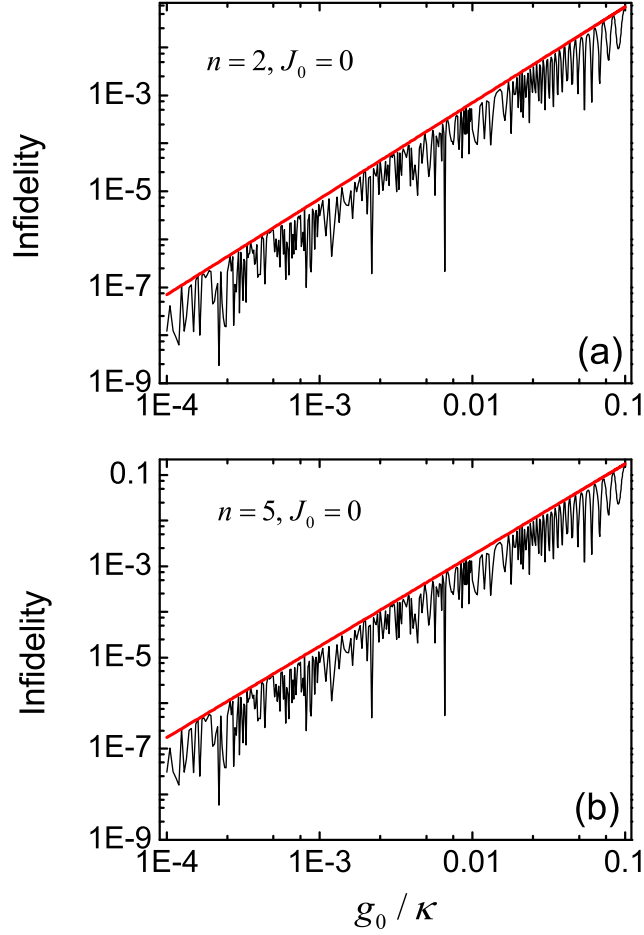


FIG. 3: (color online) Numerical results of the transmission infidelity, $\sigma_t = 1 - F_t$, as a function of g_0/κ with either (a) $n = 2$ or (b) $n = 5$ in the uncoupled case of $J_0 = 0$. The analytic upper bounds are represented by the bold red lines. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

where, for convenient, we have used $\{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$ to replace $\{|0\rangle, |z\rangle, |N+1\rangle\}$. The environment is determined by

$$H_z = \sum_{k \neq z} \varepsilon_k |k\rangle \langle k| \quad (32)$$

of having $2N$ bosonic modes, and $\varepsilon_k = \varepsilon_{-k}$. The part V_z modelling the interaction between them is

$$V_z = \sum_{k \neq z} g_k \left[|e_1\rangle \langle k| + (-1)^{k-1} |e_3\rangle \langle k| + \text{H.c.} \right]. \quad (33)$$

Assuming that the EPTC Hamiltonian can be diagonalized through a unitary transformation T , it results in $\sum_{i,j=1}^3 T_{q',i}^\dagger T_{j,q} \langle e_i | H_E | e_j \rangle = \lambda_q \delta_{q',q}$, such that

$$H_E = \sum_{q=1}^3 \lambda_q |q\rangle \langle q|, \quad (34)$$

with $|q\rangle = \sum_{i=1}^3 T_{iq} |e_i\rangle$. Similarly,

$$V_z = \sum_{k \neq z} \sum_{q=1}^3 (G_{kq} |q\rangle \langle k| + \text{H.c.}), \quad (35)$$

where

$$G_{kq} = g_k \left[T_{q,1}^\dagger + (-1)^{k-1} T_{q,3}^\dagger \right]. \quad (36)$$

By performing a first-order perturbative treatment, V_z has no effects on the eigenenergies λ_q and ε_k ; nevertheless, the eigenstates $|q\rangle$ and $|k\rangle$ are modified by

$$|\tilde{q}\rangle \simeq |q\rangle - \sum_{k \neq z} \frac{G_{kq}^*}{\varepsilon_k} |k\rangle, \quad (37)$$

and

$$|\tilde{k}\rangle \simeq |k\rangle + \sum_{q=1}^3 \frac{G_{kq}}{\varepsilon_k} |q\rangle, \quad (38)$$

respectively. Combining Eqs. (37) and (38), we find, up to second order,

$$|q\rangle \simeq \left(1 - \sum_{k \neq z} \frac{|G_{kq}|^2}{\varepsilon_k^2} \right) |\tilde{q}\rangle + \sum_{k \neq z} \frac{G_{kq}^*}{\varepsilon_k} |\tilde{k}\rangle - \sum_{k \neq z} \sum_{q' \neq q} \frac{G_{kq}^* G_{kq'}}{\varepsilon_k^2} |q'\rangle, \quad (39)$$

and hence, after an iteration, $|q\rangle$ takes the form of

$$|q\rangle \simeq |\tilde{q}\rangle + \sum_{k \neq z} \frac{G_{kq}^*}{\varepsilon_k} |\tilde{k}\rangle - \sum_{k \neq z} \sum_{q'=1}^3 \frac{G_{kq}^* G_{kq'}}{\varepsilon_k^2} |\tilde{q}'\rangle. \quad (40)$$

Since

$$\begin{aligned} \langle \tilde{q}' | \tilde{q} \rangle &= \delta_{q',q} + \sum_{k \neq z} \frac{G_{kq}^* G_{kq'}}{\varepsilon_k^2}, \\ \langle \tilde{k}' | \tilde{k} \rangle &= \delta_{k',k} + \sum_q \frac{G_{k'q}^* G_{kq}}{\varepsilon_{k'} \varepsilon_k}, \\ \langle \tilde{q} | \tilde{k} \rangle &= 0, \end{aligned} \quad (41)$$

the matrix elements of $U(t)$ in the energy space can be evaluated as

$$\langle q' | U(t) | q \rangle \simeq e^{-i\lambda_q t} \delta_{q',q} - \sum_{k \neq z} \frac{G_{kq}^* G_{kq'}}{\varepsilon_k^2} e^{-i\lambda_{q'} t} + \sum_{k \neq z} \frac{G_{kq'} G_{kq}^*}{\varepsilon_k^2} e^{-i\varepsilon_k t}. \quad (42)$$

Subsequently, after inversion back to the space spanned by $\{|e_i\rangle\}$ ($i = 1, 2, 3$), we arrive at

$$\begin{aligned} \langle e_1 | U(t) | e_3 \rangle &\simeq \langle e_1 | e^{-iH_E t} | e_3 \rangle \\ &+ \sum_{k \neq z} (-1)^{k-1} \frac{g_k^2}{\varepsilon_k^2} e^{-i\varepsilon_k t} - \sum_{k \neq z} \frac{g_k^2}{\varepsilon_k^2} \left[(-1)^{k-1} \langle e_1 | e^{-iH_E t} | e_1 \rangle + \langle e_1 | e^{-iH_E t} | e_3 \rangle \right]. \end{aligned} \quad (43)$$

Combining the evolution under H_E leads to $\langle e_1 | e^{-iH_E \tau} | e_3 \rangle = (-1)^z$ and $\langle e_1 | e^{-iH_E \tau} | e_1 \rangle = 0$, and further we find

$$M_{0,N+1} \simeq (-1)^z (1 - 2\Delta_t), \quad (44)$$

for the evolution time τ . Here, we have defined

$$\Delta_t = \sum_{k < z} \frac{g_k^2}{\varepsilon_k^2} \left[1 - (-1)^{k+z-1} \cos(\varepsilon_k \tau) \right]. \quad (45)$$

The transmission fidelity is therefore modified by

$$F_t \simeq (1 - 2\Delta_t)^{2n} \simeq 1 - 4n\Delta_t. \quad (46)$$

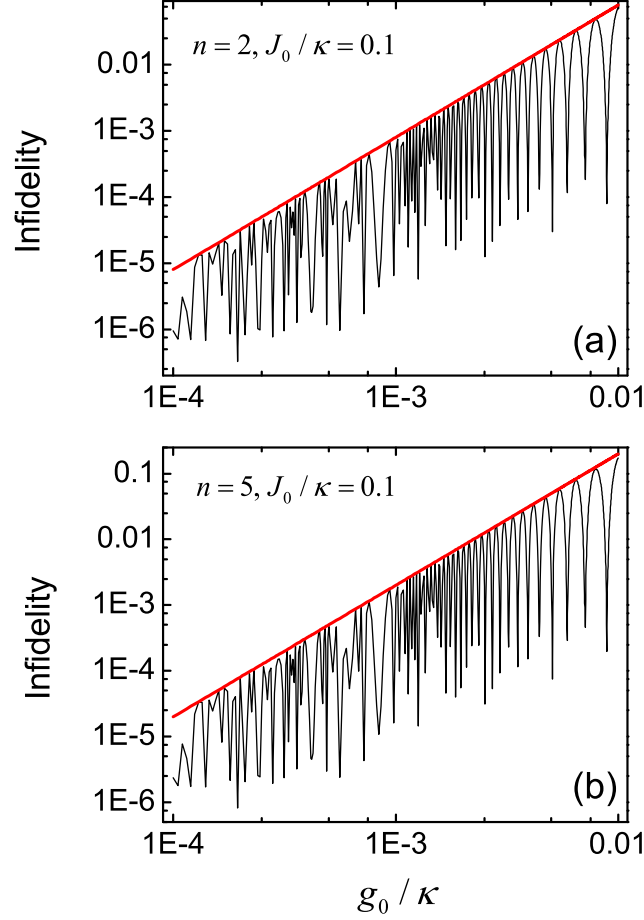


FIG. 4: (color online) Numerical results of the reflection infidelity, $\sigma_r = 1 - F_r$, as a function of g_0/κ with either (a) $n = 2$ or (b) $n = 5$ in the coupled case of $J_0/\kappa = 0.1$. The analytic upper bounds are represented by the bold red lines. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

The leakage of quantum information can be quantified using a transmission infidelity, $\sigma_t = 1 - F_t \simeq 4n\Delta_t$, and thus has an upper bound,

$$\sigma_t \leq 8n \sum_{k < z} \frac{g_k^2}{\varepsilon_k^2}. \quad (47)$$

We now consider the coupled case. In this case, the zero-energy mode of the intermediate chain is the only state coupled to the auxiliary resonator due to $J_0 \ll \kappa$, which induces two new modes as previously mentioned. The two boundary resonators are coupled to such new modes in addition to the fictitious environment; however, this coupling to the environment can be neglected so long as $g_0 \ll J_0$. The Hamiltonian is therefore reduced to

$$H_S = g_z \left[|e_1\rangle\langle e_2| + (-1)^{z-1} |e_3\rangle\langle e_2| + \text{H.c.} \right] + J_z (|e_4\rangle\langle e_2| + \text{H.c.}). \quad (48)$$

Here, we have used $\{|e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\}$ to replace $\{|0\rangle, |z\rangle, |N+1\rangle, |N+2\rangle\}$ for convenient. Using $|\gamma_1\rangle = (|e_2\rangle + |e_4\rangle)/2$, $|\gamma_2\rangle = (|e_2\rangle - |e_4\rangle)/2$ and $|\pm\rangle = (|e_1\rangle \pm |e_3\rangle)/\sqrt{2}$, the Hamiltonian H_S of Eq. (48) is brought to

$$H_S = \frac{g_z}{2} \sum_{k=1,2} \left[(|+\rangle + |-\rangle)\langle\gamma_k| + (-1)^{z-1} (|+\rangle - |-\rangle)\langle\gamma_k| + \text{H.c.} \right] + \sum_{k=1,2} J_k |\gamma_k\rangle\langle\gamma_k|, \quad (49)$$

with $J_1 = -J_2 = J_z$. When $(-1)^{z-1} = 1$, we get

$$H_S = g_z \sum_{k=1,2} (|+\rangle\langle\gamma_k| + \text{H.c.}) + \sum_{k=1,2} J_k |\gamma_k\rangle\langle\gamma_k|, \quad (50)$$

from which we can follow the same recipe as described in the uncoupled case to obtain

$$M_{0,0} \simeq 1 - 2\Delta_r, \quad (51)$$

where

$$\Delta_r = \frac{g_z^2}{2J_z^2} [1 - \cos(J_z t)]. \quad (52)$$

For $(-1)^{z-1} = -1$, it has the same result as mentioned in Eq. (51). In direct analogy to the uncoupled case, the reflection fidelity is modified by

$$F_r \simeq (1 - 2\Delta_r)^{2n} \simeq 1 - 4n\Delta_r. \quad (53)$$

Correspondingly, the reflection infidelity is $\sigma_r = 1 - F_r \simeq 4n\Delta_r$, together with an upper bound,

$$\sigma_r \leq 4n \frac{g_z^2}{J_z^2}. \quad (54)$$

To confirm our calculation of quantum information leaking into the off-resonant couplings [54], we compare numerical results of the transmission infidelity to the analytical upper bound as depicted in Fig. 3, as well as the reflection infidelity in Fig. 4. It is found that this upper bound is in excellent agreement with the numerical results. In addition, the leakage of quantum information decreases with decreasing g_0/κ , so that this leakage can, in principle, be made arbitrarily small.

V. CONCLUSIONS

In this paper, we have proposed a new approach for multiphoton controllable transport between two remote resonators being coupled to the ends of a coupled-resonator chain of arbitrary length. This manner essentially enables a coherent long-range interaction between the two spatially separated resonators, in this case, the pure Hamiltonian evolution for a specific time is referred to as a swap operation of the two boundary resonators. As a result, an arbitrary multiphoton quantum state can be transported with quantum information leakage arbitrarily close to zero. However, if an auxiliary resonator is harnessed to coupled to one cavity of the intermediate chain, this coherent long-range interaction will be eliminated, so the two boundary resonators are decoupled from the intermediate chain, yielding that the time evolution functions as an identity operation. Thus, an arbitrary multiphoton quantum state can be reflected back with quantum information leakage also arbitrarily close to zero. Our approach potentially allows for realizing controllable transport of an arbitrary dimensional quantum state or even a coherent state [see the Appendix], and can also be directly generalized to quantum networks consisting of at least three quantum registers. In fact, although we have discussed the specific case of a coupled-resonator chain, such a description is consistent for quantum coupled spin systems of having been widely studied [55–62]. Hence, the present approach could be applied to the realization of scalable quantum devices, for example, quantum switches.

VI. ACKNOWLEDGMENTS

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Appendix

In this appendix, we will discuss the controllable transport of an arbitrary d -dimensional multiphoton state $|\psi\rangle$, which is a linear superposition of Fock states, $|\psi\rangle = \sum_{n=0}^{d-1} \alpha_n |n\rangle$. When preparing this state in the left resonator, the initial state of the total system then is

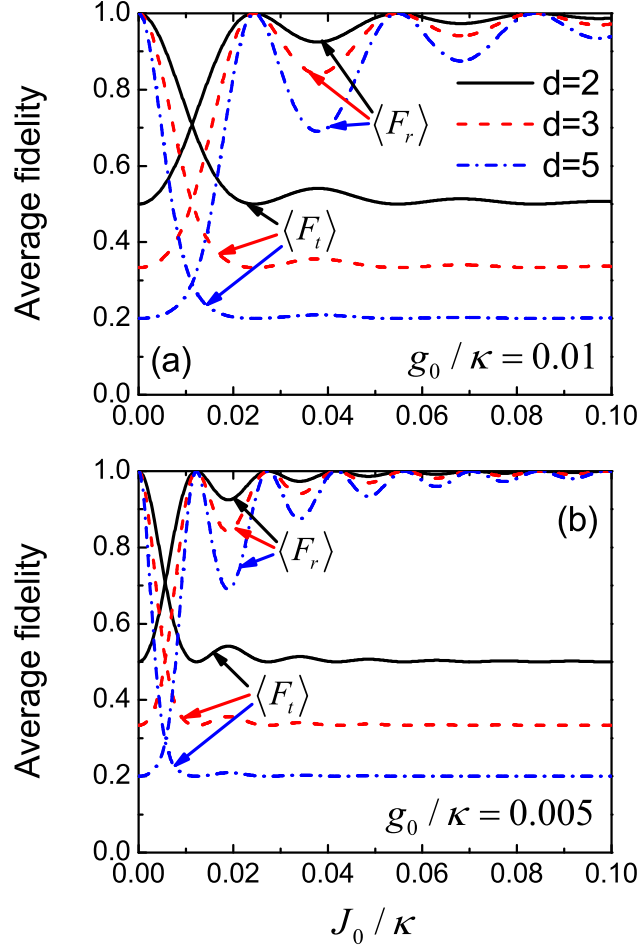


FIG. A1: (color online) Average transmission fidelity $\langle F_t \rangle$ and average reflection fidelity $\langle F_r \rangle$ as a function of J_0/κ with either (a) $g_0/\kappa = 0.01$ or (b) 0.005 for $d = 2, 3$ and 5 , corresponding to solid black lines, dashed red lines and dot-dashed blue lines, respectively. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

$$|\Phi(0)\rangle = |\psi\rangle|0\rangle|0\rangle_{N+1} = \sum_{n=0}^{d-1} \frac{\alpha_n (c_0^\dagger)^n}{\sqrt{n!}} |0\rangle_T. \quad (\text{A1})$$

Under the total Hamiltonian H_T , according to Eq. (21), the time evolution results in

$$|\Phi(t)\rangle = \sum_{n=0}^{d-1} \sum_{r=0}^n \alpha_n f_\mu(r, n) |r\rangle_{\delta_\mu} |n-r\rangle_\mu, \quad (\text{A2})$$

so the corresponding density matrix could be expressed as

$$\rho(t) = \sum_{n, n'=0}^{d-1} \sum_{r=0}^n \sum_{r'=0}^{n'} \alpha_n \alpha_{n'}^* f_\mu(r, n) f_\mu^*(r', n') |n-r\rangle_\mu \langle n'-r'| \otimes |r\rangle_{\delta_\mu} \langle r'|. \quad (\text{A3})$$

Consequently, by tracing out the variables of the collective mode, the reduced density matrix of the resonator μ is evaluated as

$$\rho_\mu(t) = \sum_{n, n'=0}^{d-1} \sum_{r=0}^{\min\{n, n'\}} \alpha_n \alpha_{n'}^* f_\mu(r, n) f_\mu^*(r, n') |n-r\rangle_\mu \langle n'-r|. \quad (\text{A4})$$

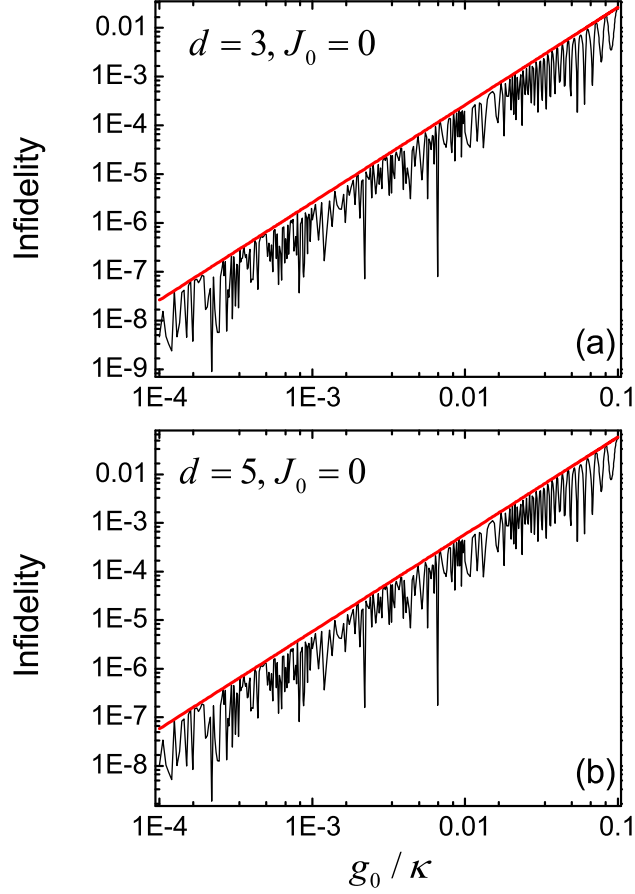


FIG. A2: (color online) Numerical results of the average transmission infidelity, $\langle \sigma_t \rangle = 1 - \langle F_t \rangle$, as a function of g_0/κ with either (a) $d = 3$ or (b) $d = 5$ in the uncoupled case of $J_0 = 0$. The analytic upper bounds are represented by the bold red lines. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

After a straightforward computation, the transmission and reflection fidelities are given, respectively, by

$$F_t \equiv \langle \psi | P^\dagger \rho_{N+1}(\tau) P | \psi \rangle = \sum_{n,n'=0}^{d-1} \sum_{r=0}^{\min\{n,n'\}} (-1)^{(n+n')z} \alpha_n \alpha_{n'-r} \alpha_{n'}^* \alpha_{n-r}^* f_{N+1}(r, n) f_{N+1}^*(r, n'), \quad (\text{A5})$$

along with $P = \exp\{iz\pi c_{N+1}^\dagger c_{N+1}\}$ to ensure the right phase evolution, and

$$F_r \equiv \langle \psi | \rho_0(\tau) | \psi \rangle = \sum_{n,n'=0}^{d-1} \sum_{r=0}^{\min\{n,n'\}} \alpha_n \alpha_{n'-r} \alpha_{n'}^* \alpha_{n-r}^* f_0(r, n) f_0^*(r, n'). \quad (\text{A6})$$

In order to quantify information transport more precisely, we need to calculate the average transmission and reflection fidelities over all initial pure states. Characterizing a d -dimensional pure state by means of the Hurwitz-parametrization method [63, 64], then yields

$$\langle F_t \rangle = \frac{1}{d(d+1)} \left[\sum_{n=0}^{d-1} \sum_{r=0}^n |f_{N+1}(r, n)|^2 + \sum_{n,n'=0}^{d-1} (-1)^{(n+n')z} f_{N+1}(0, n) f_{N+1}^*(0, n') \right] \quad (\text{A7})$$

and

$$\langle F_r \rangle = \frac{1}{d(d+1)} \left[\sum_{n=0}^{d-1} \sum_{r=0}^n |f_0(r, n)|^2 + \sum_{n,n'=0}^{d-1} f_0(0, n) f_0^*(0, n') \right]. \quad (\text{A8})$$

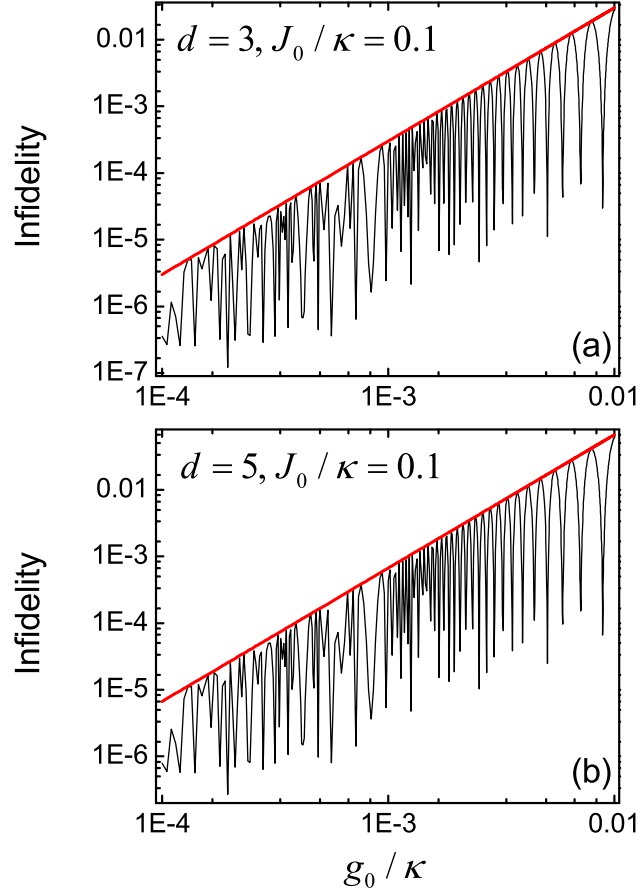


FIG. A3: (color online) Numerical results of the average reflection infidelity, $\langle\sigma_r\rangle = 1 - \langle F_r\rangle$, as a function of g_0/κ with either (a) $d = 3$ or (b) $d = 5$ in the coupled case of $J_0/\kappa = 0.1$. The analytic upper bounds are represented by the bold red lines. Here, the evolution time $t = \tau$ and we choose $N = 7$, $m = 3$.

In Fig. A1 we plot the average transmission fidelity and the average reflection fidelity. It is found that they exhibit similar behaviors to what we have observed in Fig. 2, except that, in the limit $J_0 \ll g_0$, the average transmission fidelity is close to $1/d$ instead. This difference arises from the fact that the vacuum component of the superposition state $|\psi\rangle$ has an average population probability of $1/d$ after we average over all initial pure states. Hence, controllable transport can be achieved not only for the Fock states but also for arbitrary multiphoton quantum states.

We now consider the quantum information leakage. Specifically, we use an average transmission infidelity $\langle\sigma_t\rangle = 1 - \langle F_t\rangle$ for the uncoupled case, and an average reflection infidelity $\langle\sigma_r\rangle = 1 - \langle F_r\rangle$ for the coupled case, to quantify the quantum information leakage. It follows, on combining Eqs. (22), (45) and (A7), that the average transmission infidelity $\langle\sigma_t\rangle$ is

$$\langle\sigma_t\rangle = \frac{2d(d-1)}{d+1}\Delta_t, \quad (\text{A9})$$

thereby having an upper bound,

$$\langle\sigma_t\rangle \leq \frac{4d(d-1)}{d+1} \sum_{k < z} \frac{g_k^2}{\varepsilon_k^2}. \quad (\text{A10})$$

In a similar way, the average reflection infidelity $\langle\sigma_r\rangle$ is

$$\langle\sigma_r\rangle = \frac{2d(d-1)}{d+1}\Delta_r, \quad (\text{A11})$$

and also has an upper bound

$$\langle \sigma_r \rangle \leq \frac{2d(d-1)}{d+1} \frac{g_z^2}{J_z^2}. \quad (\text{A12})$$

As described in the main text, we compare numerical results of the average transmission and reflection infidelities to their analytical upper bounds as shown in Figs. A2 and A3, respectively. We find, as expected, that the quantum information leakage for controllable transport of arbitrary multiphoton states is limited by two upper bounds which can be made arbitrary small, in direct analogy to the cases of Fock states in Figs. 3 and 4.

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